National Aeronautics and Space Administration

NASA Explorer Schools Pre-Algebra Unit

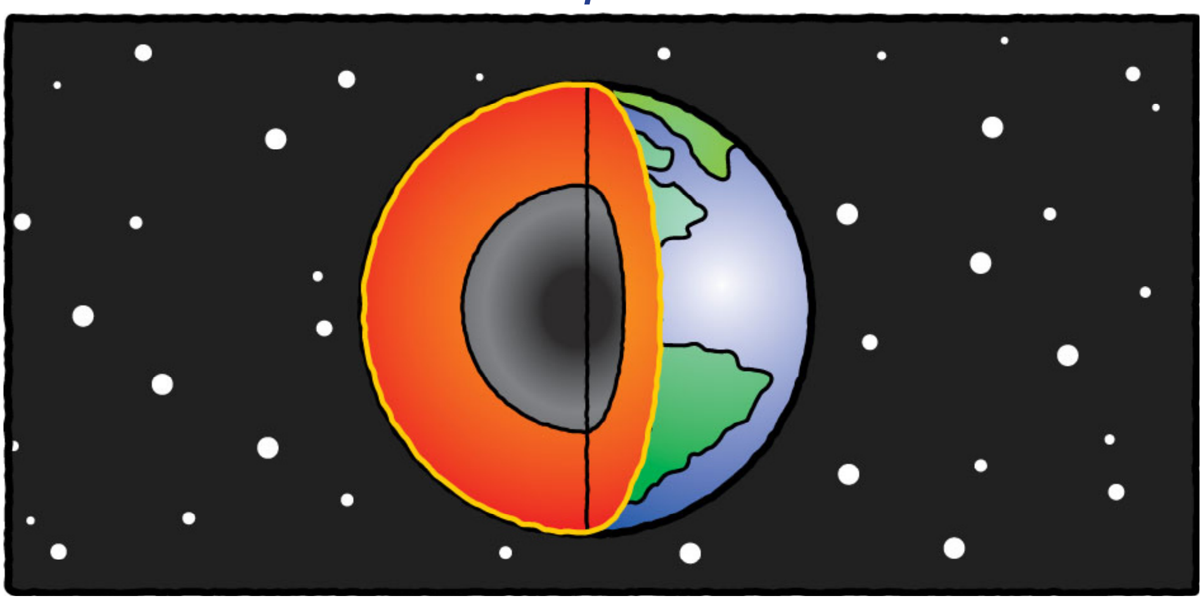
Lesson 2 Student Workbook

ANSWER GUIDE

Solar System Math

Comparing Mass, Gravity, Composition, & Density

What range of values for the mass of a planet or moon has surface conditions suitable for human exploration?



http://quest.nasa.gov

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Name: ______________________________ Date: ________________

Pre-Lesson Activity

Make Observations!

1. *Without touching them*, look at the three containers and notice their similarities and differences. List your observations below: same size same shape different colors	2. Now observe the containers more closely by touching them and moving them, but *do not open them*. Write your new observations below: different weight different temperature, sound, etc same texture

3. Measure the *weight* of each of the three containers. Be sure to record your unit of measurement. Answers will vary but should be recorded as ounces, pounds, grams, kilograms, etc. If possible, use a triple-beam balance to measure the weight in grams.

   ________ Container A   ________ Container B   ________ Container C

4. Using a *single finger*, try moving each container one at a time. Record which container is easiest and which is hardest to move. Answers will vary based on the contents of the containers; however, the container with air should be the easiest to move.

   ________ Easiest   ________ Medium   ________ Hardest

5. Imagine you are on a spacecraft orbiting Earth with the same containers. If you had to move these containers with just one finger, *which container* (if any) *would be the most difficult to move and which would be the easiest?* Answers will be the same as in question 4, with the air-filled container being the easiest to move.

   ________ Easiest   ________ Medium   ________ Hardest

Name: ______________________________ Date: ________________

Mass vs. Weight
MASS is the amount of “stuff” (MATTER) inside an object.
WEIGHT is a measurement of the force of gravity on the mass of an object.

6. Would the *weight* of these three containers change in space?

   Yes, because the containers would be nearly weightless in microgravity.

   No, because ______________________________

7. Would the *mass* of these three containers change in space?

   Yes, because ______________________________

   No, because the mass of the containers would remain the same.

Draw Conclusions!

8. Objects that have the same size and shape may weigh different amounts and have different masses based on their matter or composition.

9. An object’s weight will (change / stay the same) in space and on other planets, and its mass will (change / stay the same) in space and on other planets.

10. Objects of greater mass are (easier / harder) to move than objects of lesser mass.

11. Statement 10 would be (true / false) in space and on other planets and moons.

Make a Guess!

12. Based on the observations you have made and the conclusions you have drawn, guess the composition (or content) of each container. Answers will vary based on the contents you selected (water, sand, air, etc).

    NOTE: Be sure students recognize that the least massive container is full of AIR—it is *not* empty!

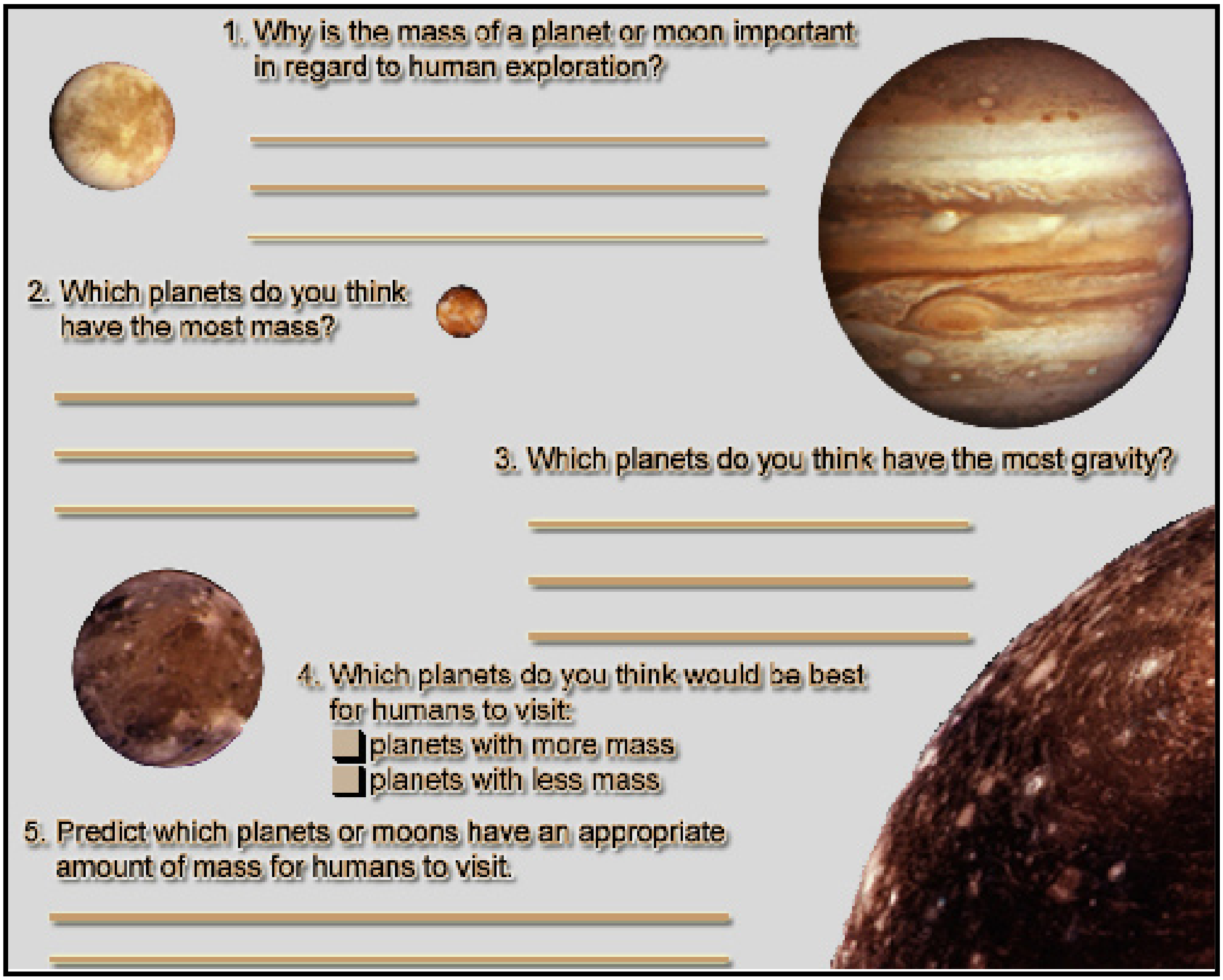
________	________	________
Container A	Container B	Container C

Name: Date: .

Planetary Mass and Human Exploration

Before you begin gathering data about the planets, you should make a hypothesis (or educated guess) as to which planets or moons have a mass that will support human exploration. Use the following questions to help you formulate your hypothesis.

Student answers will vary and at this point in the lesson, all reasonable answers should be accepted.



Name: Date: .

Minimum and Maximum Values

Imagine you are going to have a party at your house, and you are going to invite 20 guests. You need to determine a "range" of the number of pizzas that you think your guests will eat. Answers will vary, but sample responses are shown below.

1. How many pizzas do you think your guests will eat? Accept all reasonable responses.

2. What is the *minimum* (least) number of pizzas that your guests will eat? Why?

   minimum: 5 pizzas

   Less than 5 pizzas would not be enough because one pizza will serve no more than four people.

3. What is the *maximum* (greatest) number of pizzas that your guests will eat? Why?

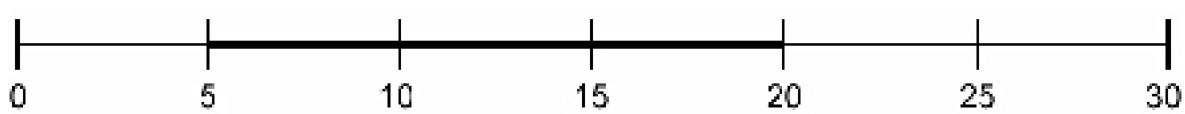
   maximum: 20 pizzas

   More than 20 is too many because it is not likely that the guests would each eat more than one whole pizza.

4. Based on your *minimum* and *maximum* numbers, what is the *range* of the number of pizzas you will need to order for your party?

$$5 \leq x \leq 20$$

5. Create a number line showing your *range* of values. Remember to label your line in uniform (equal) segments.



This sample response determined a range of values from 5 to 20 pizzas. Any number between 5 and 20, including 5 and 20, is acceptable.

Name: Date: .

Circles and Spheres

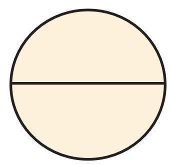
Think about the differences and similarities between circles and spheres, and then answer the questions below.

1. List three objects that are spheres. Answers will vary. (examples below)

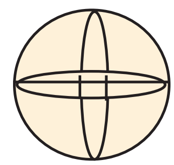
    globe   basketball   marble

2. Draw two lines: one line to indicate the *diameter* of the circle and one line to indicate the *diameter* of the sphere.

Circle

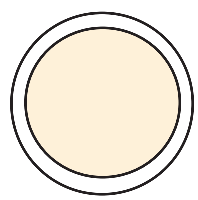


Sphere

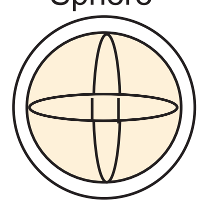


3. Draw two lines: one line to indicate the *circumference* of the circle and one line to indicate the *circumference* of the sphere.

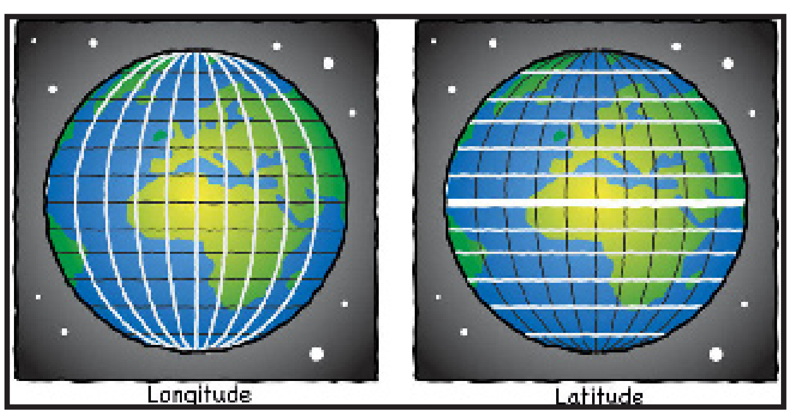
Circle



Sphere



4. A *great circle* is a circle on the surface of a sphere that divides the sphere into two equal hemispheres. Great circles have the same circumference as the sphere.



5. ALL lines of longitude are great circles.

6. ONE line of latitude is a great circle. It is called the equator.

Name: Date: .

Volume

1. What are the different sizes of:

ice cream cartons?	milk cartons?	soda bottles or cans?
pint	pint, half pint	fluid ounce
quart	quart	1 liter
gallon	gallon, half gallon	2 liter

2. How much cargo can fit into these moving vans? (You may use the Internet or a moving company's brochure to find your answer. You do not need to calculate.) Answers may vary. The answers below are from http://www.uhaul.com.

6'x12' trailer	17' moving van	26' moving van
396 cubic feet	855 cubic feet	1,592 cubic feet

3. What units of measurement are used to measure volume? List as many as you can... think about liquid measurements, dry measurements, standard measurements, and metric measurements. Answers may include:

liter	gallon	quart	pint	fluid ounce
cubic meter	cubic yard	cubic foot	cubic inch	cubic kilometer

4. What 3 dimensions are used to measure volume?

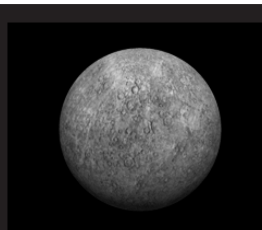
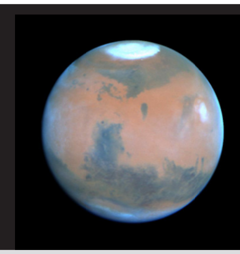
length	width	height

5. How would you determine (find out) how much space is in your classroom?

Answers may vary depending on the size and shape of the room but should include a proper unit of measurement (for example, cubic feet) and multiplying the three dimensions: L x W x H.

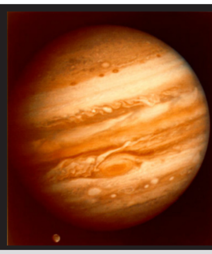
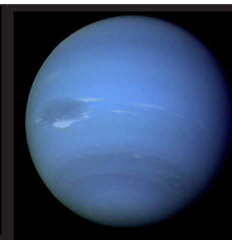
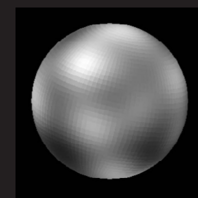
Name: Date: .

Lesson 2 Planet Data Sheet — Inner Planets

				
Planet	**Mercury**	**Venus**	**Earth**	**Mars**
Diameter in km	4,878 km	12,104 km	12,755 km	6,790 km
Circumference in km	15,329 km	38,025 km	40,075 km	21,344 km
Mass in kg	33 x 10^{22} kg	487 x 10^{22} kg	597 x 10^{22} kg	64.2 x 10^{22} kg
Density in kg/m^3	5,427 kg/m^3	5,243 kg/m^3	5,515 kg/m^3	3,933 kg/m^3
Gravity in m/s^2	3.70 m/s^2	8.87 m/s^2	9.80 m/s^2	3.71 m/s^2
Composition (rocky, frozen, gaseous, etc)	rocky	rocky	rocky	rocky
Surface fly by description	barren, no atmosphere	mountainous, harsh atmosphere	water, life, atmosphere	dry, rugged, some atmosphere

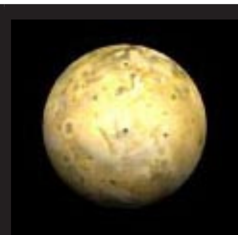
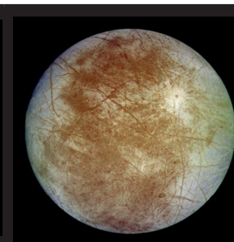
Name: Date: .

Lesson 2 Planet Data Sheet — Outer Planets

					
Planet	**Jupiter**	**Saturn**	**Uranus**	**Neptune**	**Pluto**
Diameter in km	142,796 km	120,660 km	51,118 km	49,528 km	2,300 km
Circumference in km	449,179 km	378,675 km	160,592 km	155,597 km	7,232 km
Mass in kg	190,000 x 10^{22} kg	56,800 x 10^{22} kg	8,680 x 10^{22} kg	10,200 x 10^{22} kg	1.25 x 10^{22} kg
Density in kg/m^3	1,326 kg/m^3	687 kg/m^3	1,270 kg/m^3	1,638 kg/m^3	1,750 kg/m^3
Gravity in m/s^2	24.79 m/s^2	10.44 m/s^2	8.87 m/s^2	11.15 m/s^2	0.58 m/s^2
Composition (rocky, frozen, gaseous, etc)	gaseous	gaseous	gaseous	gaseous	thin crust, methane ice
Surface flyby description	gaseous stormy	gaseous stormy	gaseous blue, cold	gaseous blue, cold	rocky, frozen no atmosphere

Name: Date: .

Lesson 2 Planet Data Sheet — Moons

					
Planet	**The Moon**	**Titan**	**Io**	**Europa**	**Triton**
Diameter in km	3,476 km	5,150 km	3,630 km	3,138 km	2,700 km
Circumference in km	10,916 km	16,171 km	11,398 km	9,583 km	8,748 km
Mass in kg	7.35×10^{22} kg	13.5×10^{22} kg	8.93×10^{22} kg	4.80×10^{22} kg	2.14×10^{22} kg
Density in kg/m^3	3,350 kg/m^3	1,881 kg/m^3	3,530 kg/m^3	3,010 kg/m^3	2,050 kg/m^3
Gravity in m/s^2	1.62 m/s^2	1.35 m/s^2	1.80 m/s^2	1.31 m/s^2	0.78 m/s^2
Composition (rocky, frozen, gaseous, etc)	rocky	crust, icy watery mantle, rocky core	crust, molten & solid mantle, iron core	watery icy crust, rocky, metal core	crust, icy methane, small core
Surface flyby description	rocky, barren, no atmosphere	rocky, barren, harsh atmos	rocky, volcanic, no atmosphere	rocky, maybe water, no atmosphere	rocky, barren, cold, frozen
Parent Planet (see "planetary symbol")	Earth	Saturn	Jupiter	Jupiter	Neptune

Name: Date: .

Scale Model: Mass

Calculate the mass of each planet and moon, and then determine the number of cotton balls that will be needed for each planet (and moon) in the scale model.

Planet / Moon	Mass in kg x 10^{22}	Actual mass in kg	Number of cotton balls based on Earth's mass
Mercury	33.0 x 10^{22} kg	3.30 x 10^{23} kg	0.055 (~ 1/20 or half of one-tenth)
Venus	487 x 10^{22} kg	4.87 x 10^{24} kg	0.816 (~ 8/10 or 4/5)
Earth	597 x 10^{22} kg	5.97 x 10^{24} kg	1.0 cotton ball
Mars	64.2 x 10^{22} kg	6.42 x 10^{23} kg	0.108 (~ 1/10)
Jupiter	190,000 x 10^{22} kg	1.90 x 10^{27} kg	318.258
Saturn	56,800 x 10^{22} kg	5.68 x 10^{26} kg	95.142
Uranus	8,680 x 10^{22} kg	8.68 x 10^{25} kg	14.539
Neptune	10,200 x 10^{22} kg	1.02 x 10^{26} kg	17.085
Pluto	1.25 x 10^{22} kg	1.25 x 10^{22} kg	0.002 (very small piece)
Moon	7.35 x 10^{22} kg	7.35 x 10^{22} kg	0.012
Titan	13.5 x 10^{22} kg	1.35 x 10^{23} kg	0.023
Io	8.93 x 10^{22} kg	8.93 x 10^{22} kg	0.015
Europa	4.80 x 10^{22} kg	4.80 x 10^{22} kg	0.008
Triton	2.14 x 10^{22} kg	2.14 x 10^{22} kg	0.004

Name: Date: .

Scale Model: Volume (using circumference)

Calculate the scale circumference of each planet and moon based on Pluto's scale circumference being equal to 1 centimeter.

Planet / Moon	Diameter in km	Circumference in km	Scale circumference in cm
Mercury	4,878 km	15,329 km	2.1 cm
Venus	12,104 km	38,025 km	5.3 cm
Earth	12,755 km	40,075 km	5.5 cm
Mars	6,790 km	21,344 km	3.0 cm
Jupiter	142,796 km	449,179 km	62.1 cm
Saturn	120,660 km	378,675 km	52.4 cm
Uranus	51,118 km	160,592 km	22.2 cm
Neptune	49,528 km	155,597 km	21.5 cm
Pluto	2,300 km	7,232 km	1.0 cm
Moon	3,476 km	10,916 km	1.5 cm
Titan	5,150 km	16,171 km	2.2 cm
Io	3,630 km	11,398 km	1.6 cm
Europa	3,138 km	9,583 km	1.3 cm
Triton	2,700 km	8,748 km	1.2 cm

Name: Date: .

Scale Model: Observations

Quietly walk around the room and observe the scale model(s) that you and your classmates created. Gently use your hands to feel the differences in the models of the planets. Read the information on the cards.

What do you notice or wonder about:

- the relationship between how the planets feel and their compositionf?
- the relationship between how the planets feel and their mass?
- the relationship between how the planets feel and their gravity?

What else do you notice or wonder? Write your observations and questions in the spaces below. Answers will vary. Sample responses are shown below.

I Notice	I Wonder
Rocky planets have more tightly-compacted cotton balls (more dense)	Do gaseous planets have a surface?
Larger planets have more mass.	Do planets with greater mass always have greater gravity?
Larger planets have more gravity.	

Based on what you have observed and learned from the scale model and the information on the cards, which planets do you think would be best for humans to visit? Why?

Answers will vary. Sample response...

I think it would be best to visit rocky planets because they have a solid surface, are closer to Earth, have a lesser mass, and have a lesser gravity.

Name: Date: .

Large Planets vs. Small Planets: Density

***Density** describes how tightly packed something is—how much matter (stuff) is in a certain amount of space.*

Estimate the density of the nine planets by *listing them from greatest density to lowest density* based on the scale model. The first one (Earth) has been done for you.

most dense

Planet	Density kg/m^3	Rocky or Gaseous	Mass $kg \times 10^{22}$
1. Earth	5,515 kg/m^3	rocky	597 $kg \times 10^{22}$
2. Mercury	5,427 kg/m^3	rocky	33 $kg \times 10^{22}$
3. Venus	5,243 kg/m^3	rocky	487 $kg \times 10^{22}$
4. Mars	3,933 kg/m^3	rocky	64.2 $kg \times 10^{22}$
5. Pluto	1,750 kg/m^3	rocky	1.48 $kg \times 10^{22}$
6. Neptune	1,638 kg/m^3	gaseous	10,200 $kg \times 10^{22}$
7. Jupiter	1,326 kg/m^3	gaseous	190,000 $kg \times 10^{22}$
8. Uranus	1,270 kg/m^3	gaseous	8,680 $kg \times 10^{22}$
9. Saturn	687 kg/m^3	gaseous	56,800 $kg \times 10^{22}$

least dense

1. Which planets have *higher* densities? X rocky ❑ gaseous
2. Which planets have *lower* densities? ❑ rocky X gaseous

Circle the 4 planets with the *greatest* mass.

3. Which planets have *higher* densities? ❑ more massive X less massive
4. Which planets have *lower* densities? X more massive ❑ less massive

Name: Date: .

Large Planets vs. Small Planets: Characteristics

You have observed the planets in the scale model in terms of mass, volume, composition, and density. Use the chart below to organize general characteristics of large planets vs. small planets.

	Large Planets	Small Planets
Mass:	more mass	less mass
Volume:	greater circumference	smaller circumference
Composition:	mostly gaseous	mostly rocky
Density:	lower density	higher density
Gravity:	more gravity	less gravity

Write your conclusions about the planets and their characteristics below.

Answers will vary but may include:

The least dense planets are composed mostly of gas.

The more dense planets are composed mostly of rock.

The more massive planets have a greater volume; however, they are less dense.

The less massive planets have a lesser volume; however, they are more dense.

The larger planets have more mass, which causes them to have more gravity.

The planets with a smaller amount of gravity are composed mostly of rock.

Name: Date: .

Lesson 2 Extension Problems

Fractions, Percents, and Planets

A pie chart is a way to represent fractions or percents of a whole. The following example calculates what percent of the total mass of the nine planets is contained in a given planet.

1. How is the mass of the solar system distributed amongst the nine planets? In other words, *what percent of the total mass of all nine planets is contained in each planet?* Be sure to include descriptions and pictures to explain how you solved the problem.

Planet	Mass (kg)	% of Total Mass
Mercury	3.30×10^{23}	0.0124%
Venus	4.87×10^{24}	0.1824%
Earth	5.97×10^{24}	0.2236%
Mars	6.42×10^{23}	0.0240%
Jupiter	1.90×10^{27}	71.1610%
Saturn	5.68×10^{26}	21.2734%
Uranus	8.68×10^{25}	3.2509%
Neptune	1.02×10^{26}	3.8202%
Pluto	1.25×10^{22}	0.0005%
Total:	$\mathbf{2.67 \times 10^{27}}$	**~ 100%**

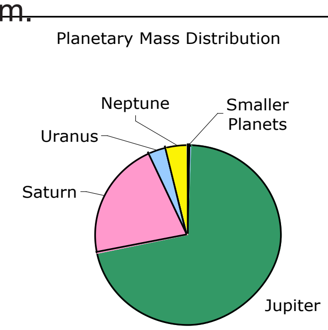
After finding solutions to the above problem, complete the following:

2. Which planets contain most of the mass of the nine planets? Jupiter & Saturn

3. If you made a pie chart showing the percent of mass for each of the nine planets, would you be able to represent the mass of the smaller planets? No

4. Find the sum of the percent of mass for the *inner planets and Pluto*. Use that as one value and use the mass of *each of the gas giants* planets to create a pie chart to represent the distribution of mass in the solar system.

Planets	% of Total Mass
Smaller planets	0.44%
Jupiter	71.16%
Saturn	21.27%
Uranus	3.25%
Neptune	3.82%



Name: Date: .

Additional Ratio and Proportion Problems

The following are problems that will require multiple steps to obtain a solution. You will need to measure lengths inside the classroom and apply what you know about scale, ratio, and proportion to solve them. You may choose the units you work with, as long as they are appropriate. Be sure to include descriptions and pictures to explain how you solved the problem.

1. An interesting relationship

Circles and spheres are very special geometric shapes. They have no corners. The formulas for the circumference and area of a circle, as well as for the surface area and volume of a sphere, are very different from the formulas for figures with straight edges like rectangles and squares. The following problem allows you to investigate an interesting relationship between two often-used measurements of any circle or any sphere.

a. Calculate the ratio of each planet's circumference to its diameter. For example, the ratio for the planet Mercury would be as follows:

$$\frac{\text{Circumference of Mercury}}{\text{Diameter of Mercury}} = \frac{15{,}329 \text{ km}}{4{,}878 \text{ km}}$$

$$= 3.14 \text{ (or } 22/7)$$

Calculate this ratio for each of the planets. Express your answers as both a fraction and as a decimal. What do you notice?

b. Measure the circumference of a great circle on a basketball. Measure the diameter of the basketball by holding a ruler against its widest point. Calculate the ratio of the circumference of the basketball to the diameter of the basketball. What do you notice? Try this for other spheres and their great circles. What is your result?

c. Look up the word "pi" on the Internet. Summarize what you learned about this very interesting number called pi (π).

For every ratio calculated for a planet's circumference to its diameter, depending on the accuracy of the measurement, the value should be approximately 3.14. The students have experimented in finding estimates for the value of pi. Pi is defined as the ratio of the circumference to the diameter of any circle.

Name: Date: .

2. A Bus to Pluto…

The mass of each planet is a measurement that is sometimes hard to conceptualize. It is helpful to compare the mass of a planet to objects that have a mass with which we are familiar. Here is an example.

a. Pluto is the least massive planet in the solar system. A public bus has a mass of approximately 14,515 kilograms. If the mass of Pluto is 125,000 kg followed by 17 zeros, estimate how many buses it would take to equal the mass of Pluto? Calculate the value (and then add 17 zeros to the end of the number!) How close was your estimate?

8.61×10^{17} buses would equal the mass of Pluto

(861,000,000,000,000,000 buses)

b. You calculated in this lesson that Pluto's mass is 0.002 of Earth's mass. Use this relationship to calculate how many buses it would take to equal the mass of Earth. (Be sure that your answer makes sense—Earth is more massive than Pluto.)

4.31×10^{20} buses would equal the mass of Earth

(431,000,000,000,000,000,000 buses)

c. If Earth's mass is equal to the mass of the number of buses you calculated in part b, estimate how many buses it would take to equal the mass of Jupiter—the most massive planet in the solar system. (You calculated in this lesson that the mass of Jupiter is approximate 318 times the mass of Earth.) Calculate the number of buses that it would take to represent the mass of Jupiter. (Be sure your answer makes sense—Jupiter is more massive than Earth.) How close was your estimate?

1.37×10^{23} buses would equal the mass of Jupiter

(137,000,000,000,000,000,000,000 buses)

Name: Date: .

3. I Gained HOW Much??!!

In this lesson, you learned that weight is affected by gravity. On another planet, an object would weigh less or more, depending on the gravity on that planet.

a. Weigh yourself here on Earth (Be careful! Think about what units you should use to record your weight and what units are on a typical scale. Remember: your mass will not change from planet to planet.)

b. For each of the other planets, use the percentage value of the other planet's gravity in terms of the Earth's gravity to calculate your weight on another planet. (For example, if Mercury has 0.4 of Earth's gravity, on Mercury you would weigh 40% of your weight on Earth.) Make a chart of what you would weigh on different planets in our solar system.

c. In addition to affecting your weight, the amount of gravity on a planet affects how high you can jump. Get a friend to measure how high you can jump standing in one spot (It might be helpful to try a few times and average your jumps.) What units of measurement will you use? This time you will use the inverse of the percentage value. To calculate the inverse, divide 1 by the decimal value of the gravity. For example, Mercury has only 40% of Earth's gravity. One divided by 0.4 is 2.5. You can jump 2.5 times higher on the surface of Mercury than you can on the surface of Earth! Make a chart showing how high you could jump on different planets in our solar system.

For a 100 lb person who can jump 30 cm on Earth standing still.

Planet/Moon	Decimal of Earth's Gravity	Weight on Planet in pounds (lbs)	Height of Jump in centimeters (cm)
Jupiter	2.5	250	12
Neptune	1.1	110	27
Saturn	1.0	100	30
Earth	1.0	100	30
Uranus	0.9	90	33
Venus	0.9	90	33
Mars	0.4	40	75
Mercury	0.4	40	75
Io	0.2	20	150
Moon	0.2	20	150
Titan	0.1	10	300
Europa	0.1	10	300
Triton	0.1	10	300
Pluto	0.06	6	500